Hydrogen in Electrodynamics III. The Hydrogen Spectrum

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Z. Naturforsch. **44a**, 504–506 (1989); received March 2, 1989

After a discussion of the one-component Schrödinger (1926) and the four-component Dirac (1928) representation of hydrogen it is shown that the six-component electrodynamic picture turns out to be considerably simpler and clearer. The computational effort is reduced to a fraction.

In the preperatory studies we proposed to approach the solution of the hydrogen problem of electrodynamics by the procedure for solving a boundary value problem.

We are given covariant source-free electrodynamics

$$\left.\begin{array}{l}\operatorname{rot}\boldsymbol{E}+\dfrac{\mu}{c}\dot{\boldsymbol{H}}=0\\ \operatorname{rot}\boldsymbol{H}-\dfrac{\varepsilon}{c}\dot{\boldsymbol{E}}=0\\ \operatorname{div}\varepsilon\boldsymbol{E}=0\\ \operatorname{div}\mu\boldsymbol{H}=0\end{array}\right\}_{\operatorname{div}\boldsymbol{E}=\operatorname{div}\boldsymbol{H}=0} \equiv \left.\begin{array}{l}\operatorname{rot}\boldsymbol{E}+\dfrac{\mu}{c}\dot{\boldsymbol{H}}=0\\ \operatorname{rot}\boldsymbol{H}-\dfrac{\varepsilon}{c}\dot{\boldsymbol{E}}=0\end{array}\right\}_{\substack{\operatorname{div}\boldsymbol{E}=\operatorname{div}\boldsymbol{H}=0\\ \boldsymbol{E}\perp\operatorname{grad}\varepsilon\\ \boldsymbol{H}\perp\operatorname{grad}\mu}} \tag{1}$$

There are eight equations with eight unknowns. We look for the six field components E_i, H_i and the interfraction $(\varepsilon_{\text{int}}^{\text{hyd}}, \mu_{\text{int}}^{\text{hyd}})$. The latter we don't want to determine by integration because of the reasons stated below [1], (25). We rather adopt them, by directly reading them off [1], (21), to be

$$\begin{aligned}(\varepsilon_{\text{int}}^{\text{hyd}}, \mu_{\text{int}}^{\text{hyd}}) &= \left(1-\frac{\Phi-m_0c^2}{\hbar\omega}, 1-\frac{\Phi+m_0c^2}{\hbar\omega}\right) \\ &= \left(1-\frac{-e_0^2/r-m_0c^2}{\hbar\omega}, 1-\frac{-e_0^2/r-m_0c^2}{\hbar\omega}\right).\end{aligned} \tag{2}$$

With (2), and for large $\hbar$, equations (1) reduce to the equations for light in vacuum – as it should be.

The quantities m_0 and e_0 we do not interpret as electron mass and charge, as explained below [1] (17). There can not be any orbiting electron within the electrodynamic H-model. We are rather dealing with a two-light field-system, in which the centers of energy of the two light fields ψ^{Re} and ψ^{Im} form a KEPLER system.

Because of the reasons mentioned between [1] (23) and [1] (26) we insert the ansatz

$$\boldsymbol{E}=\bar{\bar{E}}\,e^{-i\omega t}+\overset{+}{E}\,e^{i\omega t}, \tag{3}$$

$$\boldsymbol{H}=\bar{\bar{H}}\,e^{-i\omega t}+\overset{+}{H}\,e^{i\omega t} \tag{4}$$

into the system (1). Before that we simplify the notation by the prescription

$$\frac{\mu}{c}\to\mu,\ \frac{\varepsilon}{c}\to\varepsilon. \tag{5}$$

In addition we take into account that (1) remains invariant under the interchange

$$\boldsymbol{E}\leftrightarrow\boldsymbol{H},\ \varepsilon\leftrightarrow-\mu \tag{6}$$

so that instead of the six equations (1) we just have to solve the following three equations

$$\begin{aligned}\operatorname{rot}\boldsymbol{E}+\mu\dot{\boldsymbol{H}}&=0,\\ \operatorname{div}\boldsymbol{E}&=0,\\ \boldsymbol{E}\cdot\operatorname{grad}\varepsilon&=0.\end{aligned} \tag{7}$$

In view of the linearity of (7) we shall compute, for the sake of implicity, the two parts of (3) separately and recombine them at the end. Thus we first get, after separating out the time dependence, the two systems

$$\begin{aligned}\operatorname{rot}\overset{\pm}{E}\pm i\omega\mu\overset{\pm}{H}&=0,\\ \operatorname{div}\overset{\pm}{E}&=0,\\ \overset{\pm}{E}\cdot\operatorname{grad}\varepsilon&=0.\end{aligned} \tag{8}$$

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Writing out (8), taking into account (2) and inserting the ansatz

$$\overset{+}{E}_2 = \pm i \overset{+}{E}_1 \text{ and } \overset{+}{H}_2 = \pm i \overset{+}{H}_1 \tag{9}$$

we get for (8)

$$\begin{aligned} (\partial_1 \pm i\partial_2)\overset{+}{E}_3 &= 0, \\ \partial_3 \overset{+}{E}_3 &= \omega\mu \overset{+}{H}_3, \\ \overset{+}{E}_1 e^{\pm i\phi} \sin\theta + \overset{+}{E}_3 \cos\theta &= 0. \end{aligned} \tag{10}$$

The ansatz (9) obviously is of a form characteristic for a wave orbiting the 3-axis. We employ it for the following reasons: Firstly, we have seen in [2] that all solutions of the system (1) exhibit a preferred direction, which coincides with the 3-axis when using usual polar coordinates. Secondly, also the hydrogen-solutions of the two preliminary theories represent wave fields orbiting the 3-axis.

If in the lower system of (10) we simultaneously replace $m_k \to -m_k$ and $\phi \to -\phi$, then the lower system goes over into the upper one. From that follows

$$\bar{E}_3 = \overset{+}{E}_3 = E_3 \quad \text{and} \quad \bar{H}_3 = \overset{+}{H}_3 = H_3, \tag{11}$$

and instead of the two systems (10) we just have to solve the system

$$\begin{aligned} (\partial_1 + i\partial_2) E_3 &= 0, \\ \partial_3 E_3 &= \omega\mu H_3, \\ E_1 e^{i\phi} \sin\theta + E_3 \cos\theta &= 0 \end{aligned} \tag{12}$$

Its eigenfrequencies are determined by the first two equations. We insert the usual (simplified) separation

$$E_3 = R^E P_{l^E}^{|m_3|} e^{im_3\phi} \text{ and } H_3 = R^H P_{l^H}^{|m_3|} e^{im\phi} \tag{13}$$

and use [3] (18) and [3] (20). Omitting the signs for the absolute valves for typographic reasons and replacing m_3 by $(m_3 - 1)$ in the first equation, we obtain the system

$$\begin{aligned} &-R^E_{l^E+1} P^{m_3}_{l^E-1} + R^E_{-l^E} P^{m_3}_{l^E+1} = 0, \\ &R^E_{l^E+1} P^{m_3}_{l^E-1} (l^E + m_3) + R^E_{-l^E} P^{m_3}_{l^E+1} (l^E - m_3 + 1) \\ &= (2l^E + 1)\omega\mu R^H P^{m_3}_{l^H}. \end{aligned} \tag{14}$$

Here we may separate the spherical harmonics in two ways: either by multiplying the first equation by $-(l^E - m_3 + 1)$ or by $(l^E + m_3)$. In the first case we get the equations

$$-\omega\mu R^H + R^E_{l^E+1} = 0 \quad \text{with} \quad l^E - 1 = l^H \tag{15}$$

and in the second case

$$-\omega\mu R^H + R^E_{-l^E} = 0 \quad \text{with} \quad l^E + 1 = l^H. \tag{16}$$

Using the interchange (6) we get for (15)

$$\omega\mu R^E + R^H_{l^H+1} = 0 \qquad \text{with} \quad l^H - 1 = l^E \tag{17}$$

and for (16)

$$\omega\varepsilon R^E + R^H_{-l^H} = 0 \qquad \text{with} \quad l^H + 1 = l^E. \tag{18}$$

Out of the four differential equations (15)–(18) the first and fourth are subject to the following $l^E - l^H$ connection

$$l^{E\mathrm{I}} - 1 = l^{H\mathrm{I}} \tag{19}$$

and the second and third one to the $l^E - l^H$ connection

$$l^{E\mathrm{II}} + 1 = l^{H\mathrm{II}} \tag{20}$$

Therefore we have the two systems

$$\begin{aligned} -\omega\mu R^{H\mathrm{I}} + R^{E\mathrm{I}}_{l^{E\mathrm{I}}+1} &= 0, \\ \omega\varepsilon R^{E\mathrm{I}} + R^{H\mathrm{I}}_{-l^{H\mathrm{I}}} &= 0, \\ l^{E\mathrm{I}} - 1 &= l^{H\mathrm{I}} \end{aligned} \tag{21}$$

and

$$\begin{aligned} -\omega\mu R^{H\mathrm{II}} + R^{E\mathrm{II}}_{-l^{E\mathrm{II}}} &= 0, \\ \omega\varepsilon R^{E\mathrm{II}} + R^{H\mathrm{II}}_{l^{H\mathrm{II}}} &= 0, \\ l^{E\mathrm{II}} + 1 &= l^{H\mathrm{II}}. \end{aligned} \tag{22}$$

Simplification by fixing

$$l^{H\mathrm{I}} = l^{\mathrm{I}} \quad \text{and} \quad l^{H\mathrm{II}} = l^{\mathrm{II}} \tag{23}$$

yields

$$\begin{aligned} -\omega\mu R^{H\mathrm{I}} + R^{E\mathrm{I}}_{l^{\mathrm{I}}+2} &= 0, \\ \omega\varepsilon R^{E\mathrm{I}} + R^{H\mathrm{I}}_{-l^{\mathrm{I}}} &= 0 \end{aligned} \tag{24}$$

and

$$\begin{aligned} -\omega\mu R^{H\mathrm{II}} + R^{E\mathrm{II}}_{-l^{\mathrm{II}}+1} &= 0, \\ \omega\varepsilon R^{E\mathrm{II}} + R^{H\mathrm{II}}_{l^{\mathrm{II}}+1} &= 0. \end{aligned} \tag{25}$$

By additionally taking into account (2), (5) and [3] (17) then (24) and (25) turn into

$$\begin{aligned} -\frac{2\pi}{hc}\left(-m_0 c^2 + h\nu + \frac{e_0^2}{r}\right) R^{H\mathrm{I}} + \frac{\mathrm{d}R^{E\mathrm{I}}}{\mathrm{d}r} + \frac{l^{\mathrm{I}}+2}{r} R^{E\mathrm{I}} &= 0, \\ \frac{2\pi}{hc}\left(m_0 c^2 + h\nu + \frac{e_0^2}{r}\right) R^{E\mathrm{I}} + \frac{\mathrm{d}R^{H\mathrm{I}}}{\mathrm{d}r} + \frac{-l^{\mathrm{I}}}{r} R^{H\mathrm{I}} &= 0 \end{aligned} \tag{26}$$

and

$$-\frac{2\pi}{hc}\left(-m_0 c^2 + h\nu + \frac{e_0^2}{r}\right) R^{H\,\mathrm{II}} + \frac{\mathrm{d}R^{E\,\mathrm{II}}}{\mathrm{d}r} + \frac{-l^{\mathrm{II}}+1}{r} R^{E\,\mathrm{II}} = 0,$$

$$\frac{2\pi}{hc}\left(m_0 c^2 + h\nu + \frac{e_0^2}{r}\right) R^{E\,\mathrm{II}} + \frac{\mathrm{d}R^{H\,\mathrm{II}}}{\mathrm{d}r} + \frac{l^{\mathrm{II}}+1}{r} R^{H\,\mathrm{II}} = 0. \tag{27}$$

But these are exactly the well known systems of differential equations of the four-component preliminary theory, which yield the radial components and the hydrogen spectrum

$$\nu^{\mathrm{hyd}} = \frac{m_0 c^2}{h\sqrt{1 + \dfrac{\alpha^2}{(n_r + \sqrt{l^2 - \alpha^2})^2}}}. \tag{28}$$

Therefore we have shown that for the interfraction (2) electrodynamics (1) describes a centrally symmetric light field, which possesses the hydrogen spectrum.

The field components of the electrodynamic hydrogen field we want to examine in our next paper.

[1] H. Sallhofer, Z. Naturforsch. **43a,** 1039 (1988).
[2] H. Sallhofer, Z. Naturforsch. **39a,** 142 (1984).
[3] H. Sallhofer, Z. Naturforsch. (to appear).